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# Relative Visual Direction as a Factor in Relative Distance Perceptions

By


Walter C. Gogel
*Army Medical Research Laboratory*



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Psychological Monographs: General and Applied

# Relative Visual Direction as a Factor in Relative Distance Perceptions[1]

Walter C. Gogel

*Army Medical Research Laboratory*[2]

## Introduction

It is to be hoped that perceptions of complex visual situations can be reduced to perceptual processes which are simple in the sense that their demonstration requires only simple visual situations. The present study considers two hypotheses involving visual direction and applies these hypotheses to the prediction of the apparent path of movement of an object attached to an Ames rotating trapezoidal window. The adequacy of the two hypotheses in describing the apparent path of movement of the attached object gives some evidence of the applicability of the hypotheses to more complex visual situations than the ones from which they were developed. If applicable, the hypotheses should be useful in developing new demonstrations and in understanding the perception of movement in the visual field.

The first hypothesis to be considered is concerned with binocular vision and will be called the binocular hypothesis. A statement of this hypothesis and the experimental evidence relating to it have been given in a previous study (2). In more general form, the binocular hypothesis can be stated as follows: *The geometric binocular disparity (or stereopsis) between a test object and that object which has the smallest difference in visual direction from the test object will be the most effective stereopsis in determining the apparent distance position of the test object with respect to each of the other objects in the field of view.* For convenience in applying this hypothesis, an approximation will be made. It will be assumed that the geometric binocular disparity between the test object and a particular object will determine the apparent position of the test object when it is much closer in visual direction to the particular object than to any other object in the field.

For purposes of illustrating this hypothesis, consider the schematic top-view drawing shown in Fig. 1. The line AB represents the physical position of an object which is oriented with its left end, A, nearer to the observer than its right end, B. The object AB is viewed binocularly. Because of misleading monocular cues (for example, misleading perspective cues), AB appears to the observer to be oriented in the position of the line A′B′. The line AB might represent, for example, a top view of an Ames trapezoidal window, with the small end, A, of the window closer to the observer than the large end, B, such that the window appears to be oriented at A′B′. The long, dashed, arrowed lines represent the lines of sight from a point between the observer's eyes through A (and A′) and through B (and B′). The object shown at position $p_1$ or $p_2$ of Fig. 1 is an additional binocularly

[1] The author wishes to thank John P. Tammaro and Kay Inaba for their help in collecting and analyzing the data.

[2] The opinions or conclusions contained in the present report are those of the author. They are not to be construed as reflecting the views or endorsement of the Department of the Army.

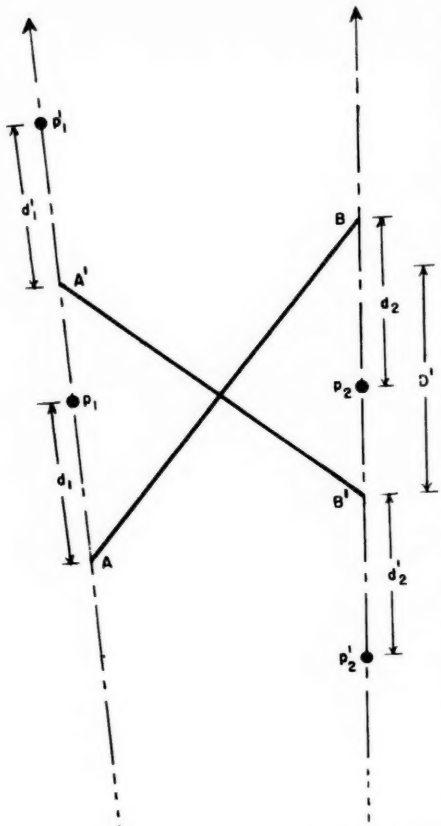


FIG. 1. Schematic representation of the difference between the physical position ($p_1$ or $p_2$) and apparent position ($p_1'$ or $p_2'$) of an object as a result of a difference between the physical and apparent position of another object AB.

observed object (for example, a small sphere). When this object is at $p_1$ it is close to the line of sight to A, and is physically a distance $d_1$ behind A. When this object is at $p_2$ it is close to the line of sight to B and is physically a distance $d_2$ in front of B. It will be assumed that the situation is such that only the stereopsis cue needs to be considered as a determining factor of the apparent relative distance position of the object at $p_1$ or $p_2$. In the terminology of the binocular hypothesis the object at $p_1$ or $p_2$ is a test object. A difference in visual direction as it is used in the binocular hypothesis is specified by drawing straight lines to the objects from a point between the observer's eyes. The angle formed between two of these lines measures the difference in the visual directions of the two objects. In Fig. 1, for example, when the test object is at $p_1$, the smallest difference in visual direction which is present occurs between this test object and A, while the largest difference in visual direction occurs between this object and B.

From the binocular hypothesis, when the test object is at $p_1$, the geometric binocular disparity between the test object and end A should determine the apparent depth position of the test object. The test object at $p_1$ should appear to be at $p_1'$, such that $d_1'$ (the apparent distance of the test object behind A') should be equal to $d_1$ (the physical distance of the test object behind A). Since $d_1$ is a physical and $d_1'$ is an apparent distance, equating $d_1$ to $d_1'$ requires some explanation. What is meant by $d_1'=d_1$ is that the perception of the depth difference between the test object at $p_1$ and the end A is not affected by the circumstance that AB appears at A'B'. In other words, the perceived distance $d_1'$ is that which would be expected from the geometric binocular disparity between $p_1$ and A.

Also, from the binocular hypothesis, the apparent depth position of the test object at $p_1$ with respect to end B should be determined by the geometric binocular disparity between the test object and the end A. But the physical and apparent orientations of AB are not in agreement. Therefore, the physical and apparent depth between the test object at $p_1$ and the end B should not be in agreement. As shown in Fig. 1, the physical position of the object

at $p_1$ is in front of the end B, but its expected apparent position $p_1'$ is in back of the apparent position (B′) of the end B.

If the test object were placed at position $p_2$ instead of $p_1$, it would be expected from the binocular hypothesis that the object would appear to be a distance $d_2'$ in front of B′ with $d_2' = d_2$. Also, the apparent position $p_2'$ of the test object at $p_2$ would be in front of the apparent position (A′) of the left end A, while physically, $p_2$ is behind A. Furthermore, if two binocularly viewed test objects were present simultaneously, one at $p_1$ and the other at $p_2$, the apparent position ($p_1'$) of the left test object would be considerably behind the apparent position ($p_2'$) of the right test object even though $p_1$ and $p_2$ were physically at the same distance from the observer.

In the situation illustrated by Fig. 1, the binocularly observed object AB appeared at A′B′. This will be called a binocular illusion. If the physical distance is known between the test object and that part of the binocular illusion which has approximately the same visual direction as the test object, it can be predicted where the test object will be seen in relation to this portion of the illusion. Also, the apparent depth relation between the test object and other parts of the illusion can be predicted. For example, in Fig. 1 if the test object at $p_1$ is seen to be a distance $d_1' = d_1$ behind A′, and A′ appears to be a distance D′ behind B′, then the test object at $p_1$ should appear to be a distance $d_1' + D'$ behind B′.

Although the situation illustrated by Fig. 1 involves a binocular illusion, it is not meant that the binocular hypothesis applies only to illusion situations. The binocular hypothesis should also apply to the case in which the physical and apparent orientation of AB are identical. In this latter case, however, there would be little or no difference between the physical positions and the apparent positions of the test object at $p_1$ or $p_2$ with respect to either A or B and, therefore, the binocular hypothesis would not manifest itself by an error in the perceived relative position of $p_1$ or $p_2$ with respect to either A or B.

Phenomena relating to the second hypothesis are most readily demonstrated when part or all of the visual field is monocularly viewed. This hypothesis will be called the monocular hypothesis and is stated as follows: *There is a tendency for an observer to see objects as being equally distant from himself, with the strength of this tendency being inversely related to the difference between the lateral visual directions of the objects (as measured from the observing eye).* Data concerning this hypothesis can be found in a previous study (3).

Figure 1 can also be used to illustrate the monocular hypothesis. Let AB and the test object at $p_1$ (or $p_2$) be observed monocularly. Let AB appear to be oriented at A′B′ (whether or not it is physically located at A′B′ is not important). Suppose that all factors which would indicate the physical depth position of the test object are severely restricted or eliminated. The monocular hypothesis would predict that a test object located anywhere along a line of sight in the directional vicinity of A (the left, long-arrowed line) would appear to be in the depth vicinity of A′, while a test object located anywhere along a line of sight in the directional vicinity of B would be seen in the depth vicinity of B′. If a monocular test object which was physically located at $p_1$ were moved in a straight line to $p_2$, it should appear to move from A′ to B′. It will be seen that this is different from the case in which everything is viewed binocularly, since with binocular observation it would be expected that a physical movement of the test object from $p_1$ to $p_2$ would result in an apparent movement from a position ($p_1'$) behind A′ to a position ($p_2'$) in front of B′.

If strong factors which specify the physical position of the test object are present, the effectiveness of the tendency expressed by the monocular hypothesis will be restricted. For this reason, either or both the test object and the other objects are to be viewed monocularly, i.e., the stereopsis cue between the test object and the other objects should not be present. The adjective *monocular* in the monocular hypothesis means that strong cues to the distance position of the test object should be reduced or eliminated. But the adjective *binocular* in the binocular hypothesis means that it is essential in using this hypothesis that stereopsis cues be present between the test object and other objects in the field of view. In this sense, binocular observation is more an integral part of the binocular hypothesis than monocular observation is of the monocular hypothesis.

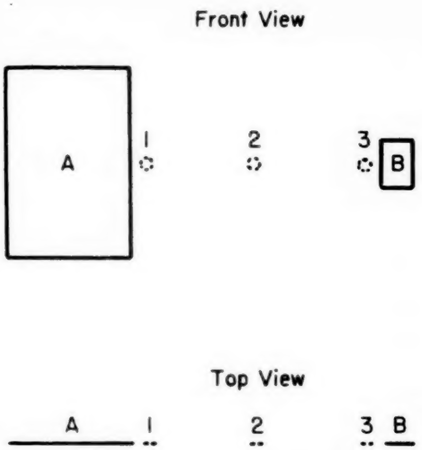


FIG. 2. Conditions for investigating the apparent depth position of a disc as a function of its lateral position between two apparently non-equidistant objects A and B.

## An Experimental Demonstration of the Two Hypotheses

In Fig. 1 the object AB extended from A to B. Therefore, if the test object were moved from $p_1$ to $p_2$, at every instant there would be some portion of the object AB which had approximately the same visual direction as the test object. This need not always occur. Consider, for example, the situation in which there is one object at A and another at B with an empty visual field in between. In this case, when the test object is moved from $p_1$ to $p_2$, the line of sight to the test object often will not be even approximately close to the line of sight to any other object. To accumulate evidence relating the two hypotheses to this type of situation and to further illustrate the hypotheses, the results from an experiment using two laterally separated playing cards are presented.

### *Apparatus*

The experimental situation used is illustrated by Fig. 2. Two seven-of-spades playing cards were presented at the same time at a distance of ten feet from the subject. The card on the left (Object A in Fig. 2) was a double-sized playing card (7 by 4 1/2 inches), and the card on the right (Object B in Fig. 2) was a half-sized playing card (1 3/4 by 1 1/8 inches). The centers of the two cards and the subjects' eyes were all at the same height from the floor. The inner edges of the two cards were laterally separated by 9 inches. A small white disc (3/8 inches in diameter) was placed between the two cards, with the center of the disc at the same height as the center of the cards. The disc was placed in one of three lateral positions. These alternative positions are indicated by the dotted lines of Fig. 2. The center of the disc was either 9/16 inch from the right edge of the left card (Position 1 of Fig. 2), 4 1/2 inches from the right edge of the left card and the left edge of the right card (Position 2 of Fig. 2), or 9/16 inch from the left edge of the right card (Position 3 of Fig. 2). The distance of the disc from the subject in all

three lateral positions was the same as that of the cards. The brightness of the disc in each of the three lateral positions was equated to that of the white portions of the playing cards (2.2 foot-lamberts as measured with a Macbeth Illuminometer). No objects were visible except the disc and the two playing cards. The remainder of the room was in darkness.

Sometimes monocular, and at other times binocular, vision was used for viewing the two cards and the disc. When binocular vision was used, the disc and both of the cards were seen by the subject with both eyes. When monocular vision was used, the disc and both cards were seen by the subject with only the right eye.

### *Procedure*

Twelve men were used as subjects. Each subject was presented with the disc in each of the three lateral positions under each of the two viewing conditions (monocular or binocular viewing). The task of the subject was to estimate in inches the apparent depth difference between the cards and the apparent depth position of the disc with respect to each of the cards. The order of presenting the various situations was varied between subjects with respect to whether the binocular or monocular view was presented first, and the order of presenting the three lateral positions of the disc.

### *Results*

The average report of the subjects was that the right card was 18 inches behind the left card with binocular viewing, and 36 inches behind the left card with monocular viewing. The average reports involving the disc and the standard deviations of these reports are given in Table 1. In computing these summarized results, the card, with respect to which the distance of the disc was being judged, determined the zero position of the response. By this it is meant that with the disc in Position 3, for example, if one subject stated that the disc appeared 4 inches in front of the right card and another subject stated that the disc appeared 2 inches behind the right card, the average of these two reports would be that the disc appeared 1 inch in front of the right card. Each mean and standard deviation in Table 1 is determined from the reports of the same twelve subjects.

The binocular results of Table 1 involve the binocular hypothesis. Following the terminology of the binocular hypothesis, the disc was the test object. The binocular illusion was produced by the misleading size cue between the binocularly observed playing cards. The size cue was such that the small (right) card appeared farther from the subject than the large (left) card, even though both cards were physically at the same distance from the subject. From

TABLE 1

MEANS AND STANDARD DEVIATIONS IN INCHES OF THE APPARENT DEPTH POSITION OF A DISC AS A FUNCTION OF ITS LATERAL POSITION BETWEEN TWO PHYSICALLY EQUIDISTANT BUT APPARENTLY NONEQUIDISTANT PLAYING CARDS A AND B

| Position of Disc | Binocular | | | | Monocular | | | |
|---|---|---|---|---|---|---|---|---|
| | Behind A | | In Front of B | | Behind A | | In Front of B | |
| | Mean | *SD* | Mean | *SD* | Mean | *SD* | Mean | *SD* |
| 1 | 0 | 4 | 20 | 25 | 16 | 32 | 21 | 36 |
| 2 | 8 | 5 | 8 | 9 | 22 | 20 | 15 | 21 |
| 3 | 18 | 17 | 1 | 5 | 40 | 36 | −4 | 18 |

Table 1, as the disc was moved from a directional position close to Card A (Position 1) to a directional position close to Card B (Position 3), the results changed from an average report of no depth difference from A and 20 inches in front of B to an average report of 18 inches behind A and 1 inch in front of B. Since the disc (the test object) was physically always at the same distance from the subject as Cards A and B, the results indicate that the physical depth position of the disc with respect to a particular card (or the geometric binocular disparity between the disc and that card) was most correctly perceived when the smallest difference in visual direction occurred between the disc and the particular card. When the disc was laterally midway between the two cards (Position 2), it appeared midway between the two cards in depth. These results suggest that a simple relationship exists between the apparent depth position of the disc and the ratio of the lateral separations of the disc from each of the cards.

The monocular results of Table 1 involve the monocular hypothesis. From the monocular hypothesis, as the difference between the lateral visual directions of the disc and a particular card decreased, the disc should have appeared to move in depth toward the apparent distance of that card. From the monocular results of Table 1, as the disc was placed laterally farther from Card A and laterally closer to Card B, the average report of the depth between the disc and a particular card increased with respect to Card A and decreased with respect to Card B. Of the three lateral positions, the position of least lateral separation of the disc from a card was the position at which the disc appeared nearest to that card in depth. One of the subjects in this monocular situation saw the disc located behind both of the cards for all three positions of the disc. If the results from this subject are removed from the monocular portion of Table 1, the mean results for this portion of the table are 6 inches, 17 inches, and 33 inches behind A for Positions 1, 2, and 3, respectively, and 28 inches, 18 inches, and 1 inch in front of B for Positions 1, 2, and 3, respectively.

With binocular vision, the apparent depth between the disc and Card A increased with each increase in the lateral separation of the disc from that card for 11 out of 12 subjects. The apparent depth between the disc and Card B increased with each increase in lateral separation of the disc from that card for 10 out of 12 subjects. The chance probability of a continuous increase in apparent depth occurring between the disc and a particular card for the two increases in lateral separation is 1/4 for a single subject. Using a p of 1/4 and a q of 3/4 in the binomial expansion, the probability that either 10 or 11 or more subjects out of 12 would see this continuous increase by chance is far less than .01. With monocular vision for 7 out of 12 subjects, the apparent depth between the disc and either card increased with each increase in the lateral separation of the disc from that card. The probability of 7 or more subjects out of 12 seeing this continuous increase by chance is less than .02.

In this experiment, the differences between the mean results from using monocular observation were similar to the differences between the mean results from using binocular observation. But, as pointed out previously, it should not be concluded that the predictions which would follow from the binocular hypothesis are always similar to the predictions which would result from the monocular hypothesis. If, for example, the disc had been placed physically more distant than the right card with lateral Position 3, it should have appeared at approximately the distance of the right card with monocular observation (the monocu-

lar hypothesis), but behind the right card with binocular observation (the binocular hypothesis).

### Application to a Complex Visual Situation

The situation to which these two hypotheses were applied experimentally is the path of apparent motion of a small object attached to the Ames rotating trapezoidal window. Ames (1) found that if a window which was made in the form of a trapezoid was rotated, it appeared, with monocular observation, to oscillate back and forth. The large end of the window always appeared to be nearer than the small end, with the result that the window appeared to reverse its direction of rotation after each movement through an apparent angle of approximately 100 degrees. He also reports that a cube attached to an end of the rotating trapezoidal window would appear to leave its point of attachment and to move in a circular path. In discussing the apparent movement of the cube, Ames remarks,

> In its rotation as the cube comes toward us, its retinal image increases, and we assume it is coming nearer to us; as it goes away from us, its retinal image becomes smaller, and we assume it is going away. . . . The variations in the sizes of the retinal images and the rate of their movement across our retina, although not uniform or constant in time, are nevertheless so translated, and the cube appears to be moving in a circular path at a relatively constant speed (1, p. 18).

Kilpatrick (4) attached a playing card to the top of the small end of the rotating trapezoidal window. In one case the card was in the plane of the window and in another case the plane of the card was perpendicular to the plane of the window. Using monocular vision the card appeared in the former case to oscillate with the window, while in the latter case "the 'real' motion of the card was perceived" (4, p. 155). Kilpatrick attributes the difference in the apparent motion of the card in these two cases to observer assumptions of "togetherness" or "apartness" of the card and window.

The apparent path of movement of an object attached to a rotating trapezoidal window can be predicted from the two hypotheses with which this paper is concerned. This is considered with the aid of Fig. 3. The solid or dotted straight lines inscribed within the circles of Fig. 3 are schematic top-view representations of a trapezoidal window. A white square is rigidly attached to the large end of the window. A top-view drawing of the square would be a short straight line in, or parallel to, the long straight line representing the window. Throughout Fig. 3 the square is drawn as a square in order to distinguish it from the window. Actually, the squares in Fig. 3 represent an object whose thickness is no greater than the thickness of the window. The letters L and S stand for the large and small end of the window, respectively. The physical positions of the trapezoidal window and the square are indicated by the solid straight lines and filled-in squares, respectively. The apparent positions of the window and square for each physical position shown directly above it are indicated by the dashed lines and outline squares, respectively. The positions of the window are either those shown by Ames (1, p. 6), or are an interpolation between several of the positions shown by Ames. The left half of Fig. 3 is concerned with binocular observation of the window and square, and the right half with monocular observation of the window and square. If the position of the observer were indicated in Fig. 3, it would be shown below each top-view drawing.

A two-dimensional object can be attached to the trapezoidal window so that its visual direction with respect to each part of the window changes as the window is rotated. A situation of this type is shown in the upper half of Fig. 3. The square is

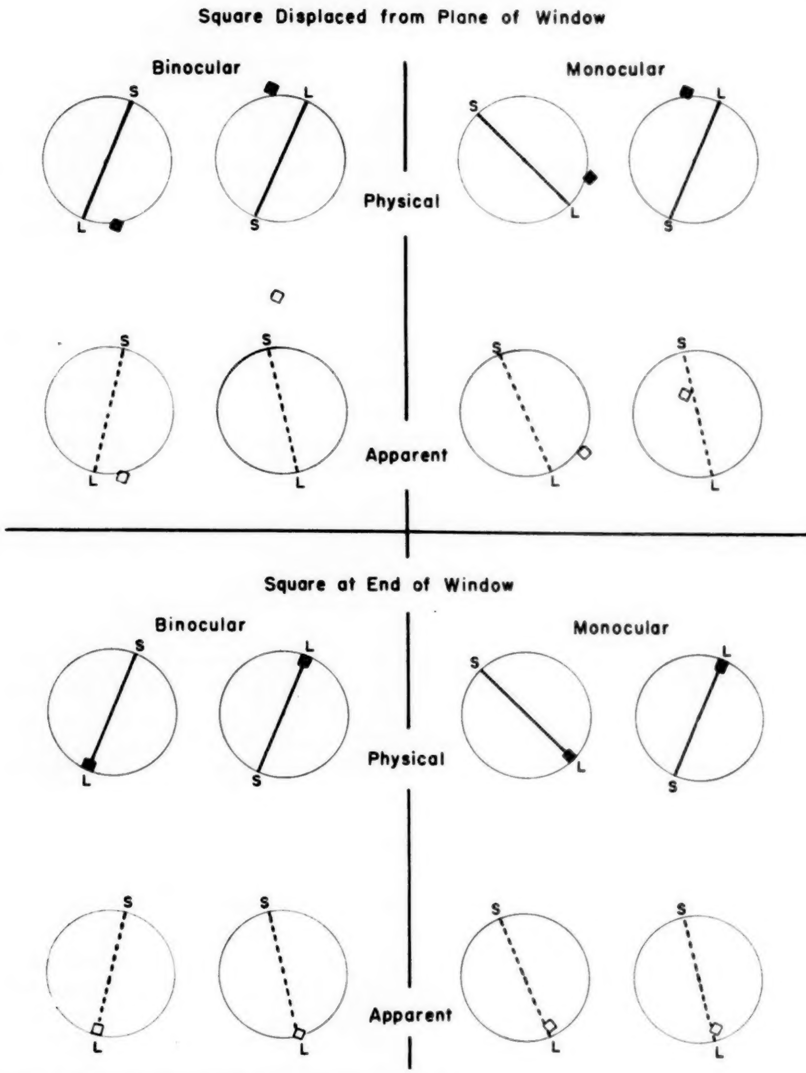


FIG. 3. Physical positions and expected apparent depth positions of a square, with two conditions of attaching the square to the trapezoidal window.

rigidly attached to the large end of the window but is displaced from the plane of the window. Or, the two-dimensional object can be attached so that, throughout the rotation of the window, there is little difference between the visual direction of the object and the visual direction of the part of the window to which the object is attached. A situation of this type is shown in the lower half of Fig. 3. The square is rigidly attached at the large end of the window and is in the plane of the window.

From the two hypotheses with which this paper is concerned, as the window rotates, the apparent path of movement of the attached square should differ, depending upon whether the part of the window which is most nearly at the same visual direction as the square is always the same part of the window or is a different part of the window at different times. Consider the upper left quadrant of Fig. 3. This quadrant involves binocular observation. In the upper left drawing of this quadrant, the large end of the window is physically in front of the small end, and the square has approximately the same visual direction as the small end. From the binocular hypothesis, in this instance there should be little or no difference between the apparent and physical position of the square with respect to the small end of the window. Since the square is physically in front of the small end by approximately one length of the window, the apparent position of the square should also be approximately one length of the window in front of the small end. But (as shown in the lower left drawing of this quadrant) the large end of the window appears closer to the observer than the small end by approximately one length of the window. Consequently, the square should appear to be in the depth vicinity of the large end. In the upper right drawing of this upper left quadrant, the square has approximately the same visual direction as the portion of the window which physically is approximately one-fourth of a window length behind the small end of the window. The square is physically a certain distance "d" behind this portion of the window. It should, by the binocular hypothesis, also appear to be distance "d" behind this portion of the window. But the small end of the window appears to be behind the large end (as shown in the lower right drawing of this quadrant). Therefore, the square, by appearing to be the distance "d" behind the portion of the window with which it has approximately the the same visual direction, should also appear to be behind the apparently far end of the window. This means that when the window rotates from the physical position shown by the upper left drawing of this upper left quadrant to the physical position shown by the upper right drawing of this quadrant, the apparent position of the square (as shown by the lower drawings of this quadrant) should move from the apparent depth vicinity of the apparently near end of the window to an apparent depth position in back of the apparently far end of the window. If the window is physically rotating counterclockwise in the top-view drawings, it will be seen that in the lower right drawing of this upper left quadrant the square is moving to the left with the small (apparently far) end of the window moving to the right. The square is about to appear to pass through the plane of the window behind the apparently far end.

The binocular hypothesis indicates that with binocular observation the attached square will be correctly perceived with respect to the portion of the window which has the same visual direction as the square. The general method of determining the predicted apparent path of movement of the square with respect to the trapezoidal window for binocular observation is as fol-

lows: (*a*) draw a top view of the apparent position of the window at a particular instant; (*b*) determine which portion of the window at this instant has most nearly the same visual direction (line of sight) as the attached object (the square); (*c*) determine the physical depth relation between this portion of the window and the square; (*d*) draw the apparent position of the square with respect to the apparent position of the window so that the physical and apparent depth relations between the square and the portion of the window most in line of sight with the square are in agreement; (*e*) repeat this procedure for successive physical positions of the trapezoidal window and attached square; (*f*) connect the successive apparent positions of the square to give the apparent path of movement of the square.

Consider the lower left quadrant of Fig. 3 which also involves binocular observation. The attached square is at the same visual direction as the large end of the window and would remain so throughout the rotation of the window. In this case, according to the binocular hypothesis, the physical depth position of the square should always be most correctly perceived with respect to the large end of the window. The large end of the rotating trapezoidal window appears to oscillate back and forth, and the square (as indicated by the lower drawings of this quadrant) should appear to move with the large end. Since the square should always appear at the same distance position as the physically large (apparently near) end of the window, it should not appear at any time to pass through the plane of the window behind this apparently near end.

With monocular observation, the physical depth position of the square should not necessarily be correctly perceived in relation to the portion of the window which has approximately the same visual direction. Instead, from the monocular hypothesis, the expected apparent depth position of the square would be near the apparent depth position of this portion of the window. With monocular, as with binocular observation, displacing the square from the plane of the window (see the upper right quadrant of Fig. 3) should make the square appear to vary its depth position in relation to the rotating trapezoidal window. But, as will be seen by comparing the monocular and binocular situations in the upper half of Fig. 3, the square in this position should not appear to move as far back in relation to the window with monocular as with binocular observation. With monocular observation and with the square displaced from the plane of the window, at some time during the movement the square should appear to pass through the plane of the window behind the apparently near end but not behind the apparently far end. This differs from the same physical situation using binocular observation since, with binocular observation, the square at some time should appear to pass through the plane of the window behind the apparently far end.

In the lower right quadrant of Fig. 3 (monocular observation), the square is physically in the plane of the window at the large end of the window. From the monocular hypothesis, when the window is rotated the square should appear to move (oscillate) with the large end of the apparently oscillating window (as indicated by the lower two drawings of this quadrant). With the square in the plane of the window, with either monocular or binocular observation, the square should not appear to pass through the plane of the window behind the apparently near end.

*Apparatus*

To test these predictions, a 3/4-inch white square was attached by a thin black

wire to the large end of a trapezoidal window. This window was part of a kit (Portable Model, Catalog No. RTW1) purchased from the Institute for Associated Research, Hanover, N. H. The window was physically rotated in a direction such that the end of the window on the left moved toward the observer, and the end of the window on the right moved away from the observer. Four alternate positions of the square with respect to the window were used. These positions are illustrated in Fig. 4, which shows the trapezoidal window with its plane parallel to the frontal plane of the observer. In Position A in Fig. 4 the square was in the plane of the window one-half inch above the large end.

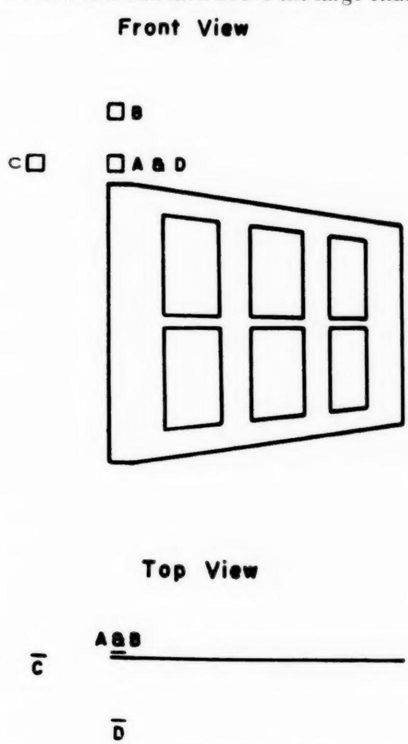


FIG. 4. Physical positions at which the square was attached to the trapezoidal window.

This position of the square is very similar to that shown in the lower part of Fig. 3. In Position B, the square was in the plane of window three inches above the large end. In Position C, the square was in the plane of the window but was displaced three inches from the large end. In Position D, the square was displaced three inches from the plane of the window. This last position of the square is very similar to that shown in the upper part of Fig. 3. These positions of the square were presented to the subjects one at a time.

The center of rotation of the trapezoidal window was placed 10 feet in front of the subjects. Dark cloth was placed in back and to the right and left of the window. The window was illuminated from below and also from the two sides with low-intensity white light, so that the window and attached square could be seen throughout the rotation. Dark baffles were placed so that none of the light sources was visible to the subjects. These baffles also were not visible to the subjects. The subject looked through two small apertures located directly in front of his eyes. These apertures restricted the visibility to the vicinity of the trapezoidal window. When the subject looked through the apertures, he saw what appeared to be a window with an adjacent square, all presented against a uniform, dark gray background. The eyes of the subject were at the height of the middle of the window. When binocular vision of the window and square was required, the subject saw the window and square through each of the two apertures. When monocular vision was required, the view through the left aperture was obscured by a black opaque card, i.e., the window and square were seen with only the right eye of the subject.

The brightness of the window when the plane of the window was parallel to the frontal plane of the subject was 1.2 foot-

lamberts, as measured with a Macbeth Illuminometer. From the viewing position of the subject, this brightness was reduced by placing a neutral density gelatin filter (Wratten ND .40) of approximately 40 per cent transmission in each of the viewing apertures. As in the demonstration of the rotating trapezoidal window by Ames (1), the axis of rotation was vertical and was in the plane of the window. The time required for a complete rotation of the window was 28 seconds.

*Procedure*

Twenty-four men were used as subjects. After preliminary observations with some of these, it was decided to have the subjects indicate the apparent path of the square by drawing a simple, top-view diagram of this path in relation to the center of the window. If the square appeared to move behind the large (apparently near) end of the window, the subjects were asked to specify the position at which the square appeared to pass through the plane of the window behind this apparently near end. Each subject was presented with the square at each of its four positions with monocular and also with binocular observation of the window and square. The order of presenting the various situations was varied between subjects with respect to whether the binocular or monocular view was presented first, and the order of presenting the four positions of the square.

*Expected Results*

As suggested by the discussion involving Fig. 3, for both monocular and binocular observation the square in Position A should appear to oscillate with the apparently near end of the window, and should not appear to pass through the plane of the window behind this apparently near end. In Position D, the square should appear to move independently of the window through part of its travel, appearing to pass through the plane of the window behind the apparently near end. But, with binocular observation, the square in Position D should appear to move farther back in relation to the apparently near end than with monocular observation. With Position B, the large end of the rotating window always has a smaller difference in visual direction in relation to the square than has any other portion of the window. Position B, with either monocular or binocular observation, should give results similar to those from position A. With Position C also, the large end of the window is always the portion of the window closest in visual direction to the square. But the square is physically sometimes in front and sometimes behind the large end and should, therefore, with binocular observation, show some independent movement in relation to this end. However, with binocular observation, the square in Position C should appear to move less far back in relation to the window than the square in Position D. It is not clear whether, with monocular observation, the path of apparent movement of the square in Position C would be expected to differ from that of Position A or B.

*Obtained Results*

For 22 of the 24 men who were used as subjects, the window appeared to oscillate through all of its physical rotation both with monocular and with binocular observation. With the remaining two men, the oscillation was momentarily disturbed at the maximum point of the illusion when binocular observation was used. The other men who reported, with binocular observation, that the apparent oscillation of the window was disturbed through an appreciable segment of the rotation were not used as subjects. The average report from the 24 subjects was that the apparent oscillation of the window extended through an angle of 100 degrees ($SD = 20°$) for mo-

TABLE 2

Means and Standard Deviations of the Portion of the Rotating Trapezoidal Window over Which the Square Appeared to Pass with the Four Conditions Illustrated by Figure 4

| Vision | A | | B | | C | | D | |
|---|---|---|---|---|---|---|---|---|
| | Mean | *SD* | Mean | *SD* | Mean | *SD* | Mean | *SD* |
| Binocular | 0 | 0 | .05 | .20 | .22 | .27 | .83 | .54 |
| Monocular | 0 | 0 | .05 | .20 | .17 | .28 | .43 | .24 |

nocular observation, and through an angle of 109 degrees ($SD=22°$) for binocular observation.

The average results and the standard deviations of these results for the apparent path of movement of the square are given in Table 2. The mean results in Table 2 are average distances expressed in units of the window at which the square appeared to pass through the plane of the window behind the apparently near end. For example, with both monocular and binocular observation, the square in Position A appeared to oscillate back and forth with the end of the window to which it was attached, and did not appear to pass through the plane of the window behind the apparently near end. This is indicated by the average reports of zero in Table 2. But in Position D with binocular observation, the square appeared to move independently of the window through part of its travel, appearing to pass through the plane of the window at an average distance of 83 per cent of the length of the window behind the apparently near end. It will be seen that the direction of the differences between the mean results from the four positions of the square are in good agreement with the predicted differences for both monocular and binocular observation. Also, as expected, the average report for Position D was larger with binocular than with monocular observation.

The *t* test was used to determine the significance of the difference between the means of Table 2. A distribution of differences was formed from the two distributions whose mean difference was to be tested for significance. The average report from binocular observation with Position D was significantly different from that with Positions A, B, or C, beyond the .001 level of confidence ($t=7.0$, 6.6, and 5.0, respectively). Also, the average report from binocular observation with Position C was significantly different from that with Positions A or B beyond the .001 and .05 level of confidence ($t=3.9$ and 2.3), respectively. The average report from monocular observation with Position D was significantly different from that with Positions A, B, or C beyond the .001 level of confidence ($t=8.6$, 7.0, and 3.9, respectively). The average report from monocular observation with Position C was significantly different from that with Positions A or B beyond the .01 and .05 levels of confidence ($t=2.9$ and 2.1), respectively. With either binocular or monocular observation, the average report from Position B was not significantly different from that from Position A at the .10 level of confidence ($t=1.2$ in both cases). The average report from Position D, with binocular observation, was significantly different from that from Position D, with monocular observation, beyond the .001 level of confidence ($t=4.3$).

### *Discussion*

A comparison between the results from using the square in Positions A and D permits an evaluation to be made of the importance of the changes in the retinal size of the square in determining the apparent path of the square. It will be noted that practically the same series of changes in the size

of the retinal image of the square occurred when the square was at either Position A or Position D. The apparent path of the square in these two positions, however, was considerably different.

A comparison between the results from using the square at Positions A and B permits an evaluation to be made of the importance of the "togetherness-apartness" factor in determining the apparent path of the square. From this factor it would be expected that the apparent path of the square in Positions A and B would differ, since the square was more separated from the window in Position B than in Position A. But this expectation is not supported by the analysis of the results.

Ames attached a three-dimensional object to the rotating trapezoidal window. Such an object attached to the rotating window so as to duplicate as nearly as possible Position A of Fig. 4 might be expected to have some differences in its apparent movement from that of the two-dimensional object used in Position A of this experiment, since a three-dimensional object extends from the plane of the window. Kilpatrick (4) attached a two-dimensional object to the rotating trapezoidal window. The playing card used by Kilpatrick behaved much like the square in Position A when the card was in the plane of the window, but not when the plane of the card was perpendicular to the plane of the window. In this latter case, portions of the card extended from the plane of the window. In both the experiments by Ames and by Kilpatrick, the object was attached to the small end of the window and, therefore, an explanation of the apparent path of the attached object when it extended from the plane of the window is probably complicated by overlay during some part of the rotation. In the present experiment, the square was attached to the large end of the trapezoidal window rather than to the small end in order to avoid having the square overlay any portion of the window during any part of the rotation.

There are some indications that factors other than those represented by the two hypotheses were operating. The average of the reports from Position D for binocular observation was not as large as expected. From the first hypothesis, the square in Position D should have appeared to move completely around (behind) the window (see the upper left quadrant of Fig. 3). Also, the average reported shape of the path of movement of the square in Position D, with monocular observation, was different from the shape which resulted when the expected path of the square was plotted using the second hypothesis. The results of this experiment, however, demonstrate that many differences between the various conditions can be successfully predicted from the two hypotheses.

Ames has indicated that, when binocular rather than monocular observation was used, the apparent oscillation of the rotating trapezoidal window was disturbed when the distance of the observer from the window was about 10 feet (1, p. 9). In the present experiment, the subjects were asked to make a simple, top-view drawing of the apparent motion of the window prior to being presented with the attached square in any of the four positions. Also, during the viewing of each of the four positions of the square, the subject was questioned about the apparent motion of the window. As mentioned previously, with 22 subjects there was no reported disturbance in the apparent oscillation of the window for either binocular or monocular observation. This difference between the results of of the two studies might be attributed to the difference in the length of the two windows used (23 1/2 inches in the study by Ames and 13 1/4 inches in the present study), or to the presence of a rectangular window above the trapezoidal window in the study by Ames but not in the present study. From the binocular hypothesis, in the study by Ames, when the ends of the rectangular window were in the directional vicinity of the corresponding ends of the trapezoidal window, there should have been some tendency to see the correct orientation of the two windows in relation to each other. Consequently, a disturbance in the apparent oscillation of the trapezoidal window might be expected.

When the physical position of the window is that shown in the upper left drawing of the upper left quadrant of Fig. 3, the amount of illusion involved in the perception of the position of the window is small. In this case, as well as for positions of the window involving larger amounts of binocular illusion, the perceived position of the square was predicted by the binocular hypothesis. If the binocular hypothesis were to be restricted so as to apply only to illusion situations, the attempt to explain the total perceived path of the attached square would involve an explanatory discontinuity as the illusion became effectively nonexistent. If only in the interest of being parsimonious, the binocular hypothesis can be considered as applying to binocular nonillusion as well as binocular illusion situations. If, for some reason, the rotating trapezoidal window had been seen in its actual (physical) orientation throughout its rotation, the attached square would have been correctly perceived with binocular observation as moving in a circular path for each of the four positions of the square which were used in this experiment. It is suggested, however, that it would move in this circular path for the same reasons that the conditions of attachment had an effect upon the apparent path of movement when the binocular illusion was present.

## Equations of Perceived Relative Distance

### *From the Binocular Hypothesis*

The binocular hypothesis specifies which stereopsis is most effective in determining

the apparent position of a test object, but it does not indicate the degree of the effectiveness. The approximation was made that when the test object was much closer in direction to one object than to another, the stereopsis between the test object and the directionally adjacent object would determine the apparent position of the test object. As illustrated by the first experiment of the paper, however, the direction of the test object might be considerably different from the direction of each of the other objects in view. A more quantitative statement is needed than that provided by the binocular hypothesis. The following development represents a first approach to such a statement.

It was suggested from the results of the first experiment of this report that, with binocular observation (see the left half of Table 1), a simple relationship exists between the apparent relative depth position of the disc and the ratio of the lateral separation of the disc from each of the playing cards. The following notation is helpful in considering this situation:

- $L$ = physical lateral separation of the two cards,
- $L'$ = apparent lateral separation of the two cards,
- $x$ = physical lateral separation of the disc and left card,
- $x'$ = apparent lateral separation of the disc and left card,
- $D$ = physical depth difference between the two cards,
- $D'$ = apparent depth difference between the two cards,
- $y$ = physical depth difference between the disc and left card,
- $y'$ = apparent depth difference between the disc and left card.

In the experiment using the two playing cards, $D=0$ and $y=0$, it will be assumed that $L=L'$ and $x=x'$, but from the experimental results, $D$ was not equal to $D'$ and $y$ was not always equal to $y'$. The binocular results in Table 1 suggest that the following equation applies to that portion of the experiment in which binocular observation was used: $y'=xD'/L$ (*Equation 1*). For example, when $x=L/2$, the disc appeared midway in depth between the two cards, i.e., $y'=D'/2$.

The general case will be considered in which neither $D$ nor $y$ are necessarily zero. This is discussed with the aid of Fig. 5. Figure 5 is a top-view drawing. If the position of the observer's eyes were indicated in this figure, it would be shown below the drawing. Points A and B represent the physical positions of two binocularly observed objects A and B. The point A′ represents the apparent relative position of A, and the point B′ represents the apparent relative position of B. Object B appears to be farther behind Object A than it actually is. This might result, for example, if A and B were similarly shaped objects with the physical size of B smaller than that of A. The point A′ and A in Fig. 5 are shown at the same position. All the error in the perception of the physically relative depth of A and B is attributed to B. This is not a necessary assumption but is made for convenience. The line connecting A and B and the line connecting A′ and B′ (or A and B′) are drawn for purposes of discussion. The point labeled "C" represents the physical position of a binocularly viewed Object C which has a different shape than A or B, so that the size cue does not occur between it and either A or B. This difference in shape is suggested by the square which is used to identify Object C. The apparent relative position of Object C is labeled C′. Two imaginary axes X and Y are placed perpendicular to each other. These axes are represented by the long solid lines. The X axis is parallel to the frontal plane of the observer and the Y axis extends away from the observer in both actual and apparent depth. The location of the intersection of these two axes is arbitrary. For convenience, it is located in Fig. 5 at Point A (or A′). Point C has the coordinate values $x$, $y$. Point C′ has the coordinate values $x'$, $y'$, or since $x$ is as-

sumed to equal $x'$, $C'$ has the coordinate values $x$, $y'$. The distance $t$ represents the physical distance between C and the line AB. The distance $t'$ represents the apparent distance between $C'$ and the line $A'B'$. Both $t$ and $t'$ will be measured from C and $C'$ in a direction perpendicular to the x axis. This simplifies the resulting equation. Actually, $t$ or $t'$ and the ordinate values of the various points should be measured along imaginary straight lines connecting the various points to the observer. But, since the distance of the observer to A, B, or C is usually considerably greater than the coordinate values of either A, B, or C as represented in Fig. 5, the above approximation should give reasonably accurate results.

In order to consider the general case, an additional assumption is required. This assumption is that for a particular physical and apparent depth relation between A and B, $t=t'$ for all values of y for any particular value of x. There is some evidence which bears upon this assumption. In a previous study (2), the binocular illusion was produced by a row of nine similar objects of differing size. Since more than two objects were used, the space between the end objects contained additional similar objects. The results of this previous study were generalized to mean that the depth position of the binocular test object was correctly perceived with respect to that object of the binocular illusion which had approximately the same visual direction as the test object. In the situation represented by Fig. 5, only two objects, A and B, are used to produce the binocular illusion. Except for the test object, C, the spatial interval between these two objects is empty. But from the binocular results of Table 1 it will be seen that, in this type of situation, the test object was correctly perceived with respect to that part of the line AB which had the same visual direction as the test object. From the binocular results of Table 1, when the disc was placed at the three different physical positions along a straight line connecting Cards A and B (see Fig. 2), it appeared to be placed along a straight line connecting the apparent positions of A and B. It might, therefore, be generalized with respect to Fig. 5 that the physical depth position of the binocular test object C will be perceived as though it were correctly seen in relation to that point on the imaginary straight line $A'B'$ which has approximately the same visual direction as the test object, i.e., $t=t'$.

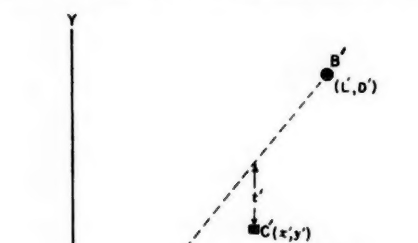


FIG. 5. Schematic representation of the difference between the physical position (C) and apparent position (C′) of an object as a result of a difference between the physical and apparent relative positions of two other objects A and B

The equation giving the apparent depth position ($y'$) of the test object (C) as a function of its physical depth position ($y$), its physical lateral position ($x$), the slope ($m$) of AB, and the slope ($m'$) of $A'B'$ is $y'=y+x(m'-m)$. This is developed as follows (see Fig. 5):

$Y$ = ordinate value of any point along AB,
$Y'$ = ordinate value of any point along $A'B'$,
$m = D/L$ = slope of AB,
$m' = D'/L'$ = slope of $A'B'$,
$t$ = physical distance perpendicular to the x axis between C and the line AB,
$t'$ = apparent distance perpendicular to the x axis between $C'$ and the line $A'B'$.

The Y value for a value of x is given by $Y = mx$.

The Y′ value for a value of x′ is given by $Y' = m'x'$.

But,

$$Y = t + y,$$

and,

$$Y' = t' + y'.$$

Assume,

$$t = t'.$$

Therefore,

$$Y - y = Y' - y'$$

and,

$$mx - y = m'x' - y'.$$

Assume,

$$x = x'$$

Therefore,

$$mx - y = m'x - y'$$

or,

$$(\textit{Equation 2})\ y' = y + x(m' - m).$$

Equation 2 contains some terms which represent physical position and others which represent apparent position. Apparent position is expressed in physical terms. For example, a measured apparent depth of 12 inches between two objects means that the apparent depth is the same as that which would occur when observing two objects, one 12 inches behind the other, in a situation where many cues to their physical relative distance are available. Equation 2 will be illustrated by an example. In Fig. 5, suppose that the imaginary line AB forms a 30-degree angle with the frontal plane of the observer ($m = \sqrt{3}/3$) but appears as though it subtended a 60-degree angle ($m' = \sqrt{3}$). Suppose, also, that Object C is located in depth 6 inches in back of Object A ($y = 6$ inches) and is laterally 10 inches to the right of A between A and B ($x = 10$ inches). The apparent distance position of C is given by $y' = 6$ inches $+ 10$ inches $(\sqrt{3} - \sqrt{3}/3)$ or $y' = 17.5$ inches. With binocular observation, Object C should appear as though it were physically located approximately 18 inches behind Object A and were correctly perceived in that position.

The previous equation $y = D'x/L$ (Equation 1) or $y' = m'x$ is a special case of Equation 2 which occurs when $y = 0$ and $m = 0$. These are the conditions for the first experiment of this report.

There are limits to the values of y, x, and m beyond which Equation 2 cannot be expected to hold. It cannot be expected to hold for objects which are so widely separated in either depth (y and D) or lateral position (x and L) that stereopsis between them is no longer an effective factor. The values of x so far considered are those between zero and L, i.e., the abscissa values of A and B in Fig. 5. If x were negative, Object C would be located to the left of Object A in Fig. 5. It would be expected from the binocular hypothesis that, in this case, some error would occur in the perception of the relative depth position of C with respect to B, and this error would be greater than that of C with respect to A. Equation 2, in this case, could be used to predict the direction of the error in the perception of C with respect to B, but it is not known whether it would successfully predict the magnitude of the error. A case similar in principle occurs when the x value of C is greater than L (the abscissa value of B). Equation 2 is expected to apply only when the value of x lies between zero and L (see Fig. 5).

Figure 5 illustrates the case in which a horizontal difference occurs in the visual direction of Objects A, B, and C. Vertical differences might also be involved. The binocular hypothesis has been stated so as to apply to differences in visual direction, regardless of the direction of these differences. Experimental evidence relating to the binocular hypothesis has been presented, however, only for the case in which the binocular illusion consisted of horizontally separated objects. Consequently, Equation 2 should be considered as applying only tentatively to situations in which vertical components of directional separation are involved in the illusion. The expectation, however, is that in such cases, Equation 2 would remain unchanged but the plane defined by the XY axes should be rotated so as to continue to contain the line AB and the line A′B′. If Point C is not in the resulting XY plane, the x value of C would probably be determined by orthogonally projecting C on this plane. The correctness of this method of determining x is suggested by the similarity of the binocular results obtained when the square was

located either directly above or three inches above the large end of the rotating trapezoidal window (Positions A and B of Fig. 4). With the window illusion occurring in the horizontal plane, vertical displacement of the square did not seem to affect the apparent path of movement of the square.

### *From the Monocular Hypothesis*

In the notation associated with Fig. 5, tentatively the monocular results of Table 1 can be described by the equation $y' = m'x$ (Equation 3). Equation 1 and Equation 3 are identical even though they involve different hypotheses. This is reflected in the similarities in the binocular and monocular results of Table 1. Equation 1 is the special case of Equation 2 which occurs when m and y are zero. Equation 3, however, is a general equation which does not involve m or y. The presence or absence of the physical terms and m and y constitutes the difference between the general equation involving the binocular hypothesis (Equation 2) and that involving the monocular hypothesis (Equation 3).

The verbal reports of the apparent depth between the two playing cards in the first experiment of this study indicate that the value of $m'$ was larger with monocular than with binocular observation. Some of the dissimilarities between the monocular and binocular results of Table 1 can be explained by this difference in $m'$. For example, for Position 2 (see Fig. 2), as expected from Equations 1 and 3, the reported depth between the disc and Card A was greater when monocular rather than binocular observation was used.

Referring to Fig. 5, it is not known whether Equation 3 will apply when x is considerably negative or considerably greater than L. As with Equation 2, data relating to Equation 3 have been gathered only for the case in which A and B were horizontally separated without a large component of vertical separation. Additional cases need to be investigated.

## Summary

An experiment was conducted to illustrate and to extend the evidence relating to two previously formulated hypotheses involving visual direction. One of the hypotheses, called the binocular hypothesis, states that *the stereopsis between a test object and that object which has the smallest difference in visual direction from the test object is the most effective stereopsis in determining the apparent distance position of the test object with respect to each of the other objects in the field of view.* The other hypothesis, called for convenience the monocular hypothesis, states that *there is a tendency to see objects as located at the same distance from the observer, with the strength of this tendency being inversely related to the difference between the lateral visual directions of the two objects.*

To test the applicability of the hypotheses to fairly complex visual situations, the two hypotheses were used in predicting the apparent path of movement of a small object attached to the Ames rotating trapezoidal window. An experiment was reported in which the trapezoidal window was viewed either monocularly or binocularly, with four conditions of attaching the small object to the window. The differences between the results from the various conditions confirm a group of predictions made from the two hypotheses.

An equation was developed from the binocular hypothesis to predict the apparrent relative depth position of a binocular object as a function of its physical relative position and the physical and apparent relative positions of other objects in the field. Also, an equation involving the monocular hypothesis was suggested.

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